

**Lines in algebraic subsets of infinite-dimensional  
projective spaces and connectedness**

E. BALLICO

**Abstract**

Let  $V$  be an infinite-dimensional vector space over a field  $K$  and  $X \subseteq P(V)$  the zero-set of finitely many homogeneous  $K$ -forms. Assume that  $K$  is either algebraically closed or a finite field. Here we prove that for all  $P, Q \in X$  there are  $K$ -lines  $D, R$  such that  $P \in D$ ,  $Q \in R$ ,  $D \cap R \neq \emptyset$  and  $D \cup R \subseteq X$ . We also give effective versions of these results when  $\dim(V)$  is finite, but large.